As promised, we discuss medians today! Conceptually, the median is very simple. It is just the middle number. Arrange all the numbers in increasing/decreasing order and the number you get right in the middle, is the median. So it is quite straight forward when you have odd number of numbers since you have a “middle” number. What about the case when you have even number of numbers? In that case, it is just the average of the two middle numbers.

Median of [2, 5, 10] is 5

Median of [3, 78, 102, 500] is (78 + 102)/2 = 90

If it’s that simple, why are we discussing it? – because it isn’t “that simple”! Conceptually it is, but when the test writers make questions using median and arithmetic mean together, they make some very mean questions! I will show you with an example, but first, we will look at a simpler question.

Question 1: A, B and C have received their Math midterm scores today. They find that the arithmetic mean of the three scores is 78. What is the median of the three scores?

(1) A scored a 73 on her exam.

(2) C scored a 78 on her exam.

Solution: Recall from the arithmetic mean post that the sum of deviations of all scores from the mean is 0.
i.e. if one score is less than mean, there has to be one score that is more than the mean.
e.g. If mean is 78, one of the following must be true:

1. All scores are equal to 78.
2. At least one score is less than 78 and at least one is greater than 78.

For example, if one score is 70 i.e. 8 less than 78, another score has to make up this deficit of 8. Therefore, there could be a score that is 86 (8 more than 78) or there could be two scores of 82 each etc.

Statement 1: A scored 73 on her exam.

For the mean to be 78, there must be at least one score higher than 78. But what exactly are the other two scores? We have no idea! Various cases are possible:

73, 78, 83 or

73, 74, 87 or

70, 73, 91 etc.

In each case, the median will be different. Hence this statement alone is not sufficient.

Statement 2: C scored 78 on her exam.

Now we know that one score is 78. Either the other two will also be 78 or one will be less than 78 and the other will be greater than 78. In either case, 78 will be the middle number and hence will be the median. This statement alone is sufficient.

Answer (B)

Were you tempted to say (C) is the answer? I hope this question shows you that median can be a little tricky. Let’s go on to the tougher question now.

Question 2: Five logs of wood have an average length of 100 cm and a median length of 116 cm. What is the maximum possible length, in cm, of the shortest piece of wood?

(A) 50
(B) 76
(C) 84
(D) 96
(E) 100

Solution:

First thing that comes to mind – median is the 3rd term out of 5 so the lengths arranged in increasing order must look like this:

___ ___ 116 ___ ___

The mean is given and we need to maximize the smallest number. Basically, the smallest number should be as close to the mean as possible. This means the greatest number should be as close to the mean as possible too (if the shortfall deviation is small, the excess deviation should by equally small).

If this doesn't make sense, think of a set with mean 20:

19, 20, 21 (smallest number is very close to mean; greatest number is very close to the mean too)
1, 20, 39 (smallest number is far away from the mean, greatest number is far away too)

Using the same logic, let's make the greater numbers as small as possible (so the smallest number can be as large as possible). The two greatest numbers should both be at least 116 (since 116 is the median). Now the lengths arranged look like this:

___ ___ 116 116 116

Since the mean is 100 and each of the 3 large numbers are already 16 more than 100 i.e. total 16*3 = 48 more than the mean (excess deviation is 48), the deviations of the two small numbers should be a total of 48 less than the mean. To make the smallest number as great as possible, each of the small numbers should be 48/2 = 24 less than the mean i.e. they both should be 76.

Answer (B).

Hopefully, it made sense to you. See you again next week for a discussion on another Statistics concept!